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Opening the black box: Finding the source of cost inefficiency

Santiago Carbó Valverde · David B. Humphrey ·
Rafael López del Paso

Abstract Parametric and nonparametric procedures are used to identify the apparent source of cost inefficiency in banking. Inefficiencies of 20–25% from earlier studies are reduced to 1–5% when, in addition to commonly specified cost function influences, variables reflecting banks' external business environment and industry indicators of ''productivity'' are added. These productivity indicators explain most of the reduction in bank operating cost over 1992–2001 and was 5 times the reduction in the dispersion of inefficiency. Inefficiency appears stable over time because it is small relative to industry-wide cost changes occurring concurrently and because technology dispersion is imperfect.

Keywords Cost efficiency · Banks

JEL Classifications G21 · G28 · E58

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S. Carbó Valverde · R. López del Paso
Departamento de Teoría e Historia Económica, Universidad de Granada, Granada, Spain

D. B. Humphrey (✉)
Department of Finance, Florida State University, Tallahassee, FL, USA
e-mail: dhumphrey@cob.fsu.edu

1 Introduction

Almost all studies of cost efficiency in banking, whether in the U.S., Europe, or elsewhere, suggest that inefficiency is relatively large and persistent. Averaging the results of 130 studies across five different types of frontier approaches for 21 countries suggests that average cost inefficiency in various nations' banking industries is 20–25% (Berger and Humphrey 1997). That is, the average bank appears to experience total operating plus interest costs that are from 20% to 25% higher than the most cost-efficient bank after controlling for: (a) differences in the value of various types of loans and securities (or deposits) in the balance sheet; (b) differences in average funding, labor, and capital costs among banks; and (c), the technology by which banking inputs are transformed into outputs.

As bank net income is around 17% of total costs (the U.S. average over 1998–2001, with lower percentages for other countries), this suggests that the average bank—not just the most inefficient among them—could more than double their profits and return on assets by restructuring their operations to look like those banks that appear to be most efficient. If true, the incentive to restructure and ''look like'' the most efficient banks—in a benchmarking or ''best practice'' context—should be very strong. Yet, the average levels of measured inefficiency do not seem to be consistently falling over time for any of the numerous countries which have been studied. Are measured inefficiencies overstated so actual incentives to improve efficiency are much weaker than they appear? Or, if correctly measured, are they largely beyond the effective control of management? And, if not overstated and not beyond management's control, what may explain these persistent differences among banks? Finally, which is more important for social or regulatory policy—determining the source of cost differences among banks at a point in time (inefficiency) or changes in costs that seem to affect all banks relatively equally over time (akin to a shift in the frontier and all bank costs)?

It is hard to answer these questions as few studies have had much success in identifying the major sources of the inefficiency being measured. Without knowing the main source(s) of the problem, it is difficult to determine why efficiency differs among banks. Unlike banking consultants who have privileged access to detailed cost data and focus on benchmarking efficiency among branches of a single bank (Sherman and Ladino 1995), existing academic studies are typically concerned with benchmarking efficiency among banks within a single country using only publicly available information. Both approaches are useful to the extent that major sources of efficiency differences can be identified.

A more comprehensive analysis to identifying bank inefficiency is used here to open the black box that has previously ''hidden'' the sources of unexplained cost inefficiency. Inefficiency associated with bank operating expenses is examined separately from inefficiency in funding costs. Cost differences among banks are then separated into their external, technical (cost function), and internal sources each with a different set of explanatory variables. With this approach, the sources of cost inefficiency are largely identified. While the sources would not surprise bank managers, we find the same variables that are the source of inefficiency are also the main source of the dramatic reduction in bank operating costs in Europe over time.

In what follows, Sect. 2 reviews studies that have attempted to identify sources of inefficiency in banking. This analysis has typically focused on measures derived from banks' balance sheets as opposed to external influences that frame a bank's economic environment or partial indicators of a bank's internal productivity that are commonly used as benchmarks within the industry. A broader set of influences is developed in this study and consist of: (1) external influences largely outside of the control of management; (2) technical influences associated with transforming banking inputs into outputs within a cost function framework; (3) influences partly under managerial control and thus are—to differing degrees—internal to the firm; and (4), a residual reflecting influences that can not be directly measured with the available data but are attributed to unknown managerial policies, organizational structure, or leadership ability.

To demonstrate the robustness of our results, in Sect. 3 we apply both a parametric model—Distribution Free Approach (DFA)—and a linear programming model—Data Envelopment Analysis (DEA). Our main parametric efficiency results are reported in Sect. 4 while those for our nonparametric model are in Sect. 5. Both are based on semi-annual observations on 46 savings banks plus 31 commercial banks in Spain over 1992–2001, giving a total panel of 1,540 observations. Efficiency estimates are presented separately for operating versus funding costs, grouped additively according to external, technical, and internal sources of efficiency, and distinguished between savings and commercial banks. Section 6 contains confidence intervals for our DFA and DEA efficiency results and indicates that although similar point estimates of efficiency are obtained, these point estimates are significantly different. Section 7 presents information showing that the apparent sources of inefficiency are also the main source of the dramatic reduction in bank operating costs in Europe over time.

To preview our conclusions in Sect. 8, most of the traditionally measured inefficiencies lie within the operating function of a bank—the funding function contains few inefficiencies. Operating cost efficiency reaches 95% with our full DFA model while a 99% level is obtained for funding expenses. This is considerably higher than the typical levels attained in more limited studies. As envisioned by the originators of the frontier efficiency concept (e.g., Charnes et al. 1978), we find that previously unexplained (residual) differences in efficiency are dominated by measures of bank productivity and are not really a ''black box'' after all. Having reduced measured inefficiency to very low levels, it is suggested that most of the remaining unexplained residual can be attributed to management decisions with a priori inherently uncertain outcomes and may best be considered as irreducible. Future analyses of efficiency differences among firms may usefully focus on productivity measures which have a significant effect on costs or profits rather than leaving these differences unexplained.

2 Determinants of inefficiency in banking

2.1 Two approaches to measuring efficiency

One approach to efficiency measurement has been to relate total banking costs to the value of various balance sheet components along with funding and labor and capital input prices within a parametric cost function. While the specific form used imposes some structure on the technical relationship between banking inputs and outputs, a more important issue is how inefficiency is measured. The composed error Stochastic Frontier Approach (SFA) typically assumes a half-normal distribution for inefficiencies in order to separate unknown inefficiencies from normally distributed error in a panel regression.[1] The Distribution Free Approach (DFA)—the model used here—assumes that averaging each bank's residuals across separate yearly cross-section regressions (in our case containing two semi-annual data sets for each year) reduces normally distributed error to minimal levels leaving only average inefficiency. Although both models involve strong assumptions, they generate similar levels and rankings of banking inefficiency (Bauer et al. 1998).

A second approach to measuring inefficiency utilizes linear programming, assumes that random error equals zero, and—unlike the cost function parametric approach—places little structure on the specification of the piecewise linear best-practice frontier that results. Of two linear programming models, Data Envelopment Analysis (DEA) is by far the most used and is used here as well.[2] While the parametric models rely on strong assumptions regarding the form of the distribution of inefficiency or the ability to average random errors to levels close to zero for individual banks over time, a limitation of the DEA model is that the more influences specified as potentially having an effect on explaining inefficiency, the lower will be the measured inefficiency. This occurs whether or not a variable is related to inefficiency since each additional influence (constraint) in the DEA approach reduces the set of banks being compared (so measured average inefficiency necessarily declines). With the DFA parametric approach, if a specified influence is unrelated to efficiency, measured inefficiency should be little affected. As there is no real consensus on which approach—parametric or non-parametric—is best, both are used to illustrate the consistency of our effort to explain inefficiency.

2.2 Previous studies determining the source of inefficiency

Studies trying to explain differences in inefficiency scores among banks have not had much success. Indeed, the resulting explanatory power of these ancillary regressions is often quite low (e.g., with $R^2s < 0.10$). Even so, a few studies have gone beyond the usual set of variables drawn from a bank's balance sheet and have been more informative. Berger and Mester (1997), for example, have expanded on the usual set of bank size and liability/asset composition variables to include organizational form, governance, market competition, geographical location, and regulatory structure. As well, Dietsch and Lozano-Vivas (2000) have looked deeper still and included variables that reflect how a bank's economic environment—regional per capita income and population, deposit, and branch density—can help explain efficiency differences between two countries. Finally, using a survey-based data set similar to a time-and-motion analysis of numerous specific retail bank deposit and loan activities, Frei et al. (2000) developed efficiency measures for 135 U.S. banks (comprising about 75% of banking assets in the early 1990s). It was suggested that these specific and diverse efficiency indicators are, when viewed in their entirety, what makes a bank efficient. If so, these micro productivity measures for individual banks should be correlated with and help ''explain'' inefficiencies measured using DFA or DEA frontier analyses. Similarly, publicly available indicators of bank productivity commonly used within the industry for inter-bank and peer group comparisons should also be able to ''explain'' these inefficiencies. Indeed, some indication of this result was found earlier for U.S. banks when including a labor/branch ratio resulted in a one-third reduction in the (previously unexplained) inefficiency residual (Berger and Humphrey 1991, p. 143).

While data availability is a contributing factor, the basic problem with most efforts to determine the main sources of efficiency differences has been a focus on balance sheet correlates with inefficiency, not on outside environmental factors or even partial measures of banking productivity common within the industry (both of which can affect costs among banking firms). We continue along the path developed by Berger and Mester, Dietsch and Lozano-Vivas, and Frei, Harker, and Hunter and find that by considering an expanded set of cost influences it is possible to reduce measured inefficiency down to very low levels—levels so low it is suggested most of the remainder can be considered inherent and irreducible.

3 Specification of two efficiency measurement models

3.1 Distribution Free Approach (DFA) to efficiency measurement

The DFA model of cost frontier measurement uses panel data but does not estimate a panel regression. Instead, for each year of the panel a separate cost function is estimated using cross-section data that relates total banking cost (TC) to observed levels of balance sheet ''output'' variables (Q_i: loans, securities) and average input prices (P_j: for funding, labor, and physical capital which can include financial capital) as in $\ln TC = C(\ln Q_i, \ \ln P_j) + \ \ln u + \ \ln v$.

[1] The assumption that most banks are close to the efficient frontier so that inefficient firms are skewed away from the frontier (as in a half-normal, Gamma, or truncated normal distribution of inefficiency) does not appear to be the case in practice (Bauer and Hancock 1993; Berger 1993). The distribution of inefficiencies is more like a symmetric normal distribution which would make it difficult to locally identify separately from normally distributed error.

[2] The other approach is the Free Disposal Hull and will be either congruent with or interior to the DEA frontier. When it is interior, lower estimates of average inefficiency will result (Tulkens, 1993).

The DFA concept of efficiency relies on the average value of the unexplained composite residual (ln u + ln v) such that for each bank over a series of (in our case 10) cross-section estimations, the random error term ln v is assumed to average out to a value close to zero while the mean value of the inefficiency term ln u (represented as ln $\bar{u}$) will reflect the average bank-specific level of cost inefficiency over the period (Berger 1993).[3] The bank with the lowest average inefficiency term ($\ln\bar{u}_{min}$) is deemed to be the most cost efficient and the efficiency of all the other i banks ($\mathrm{EFF_i}$) is determined relative to this standard:

$$\mathrm{EFF}_i = \exp(\ln \bar{u}_{\min} - \ln \bar{u}_i) = \bar{u}_{\min}/\bar{u}_i \tag{1}$$

As u_i is multiplicative to TC_i in the un-logged cost equation $TC_i = C(Q,P)_i u_i$, the ratio $\bar{u}_{\min}/\bar{u}_i$ is an estimate of the ratio of total cost of the most efficient bank, for a given scale of operation and input prices, to the total cost of bank i using the same output levels and input prices.[4] If the EFF ratio $\bar{u}_{\min}/\bar{u}_i = 0.80$, resources used at the most efficient bank represents 80% of the level of resources used at the ith bank. This suggests that the ith bank is inefficiently using around $(1.00 - 0.80)/0.80 = 0.20/0.80 = 25\ \%$ of its own resources compared to the most cost efficient bank.[5]

3.2 Data Envelopment Analysis (DEA) of efficiency

The non-parametric DEA model[6] uses linear programming to find the ''best practice'' bank in the sample ($s = 1,\ldots S$) that reflects minimum cost in producing the observed output vector (q_i) given input prices (p_j) and the technology of the cost relationship $C(q_j, p_j|V, A) = \{(q_j,p_j)$ where $q_j \leq Q\tau, x_j \leq X\tau,\ \tau \in \mathrm{R}^j_+, \sum_{j=1}^{J} \tau_j = 1\}$ and strictly satisfies the availability of outputs and input prices (denoted by A) and exhibits variable returns to scale (denoted by V). Given the technology, where τ denotes a vector of intensity variables from activity analysis, the cost performance of an individual bank j can be evaluated by comparing $j's$ observed vector of input prices p_j, incurred in producing its observed output vector q_j, with input prices (p_j) on the boundary (or best-practice frontier) of the cost set $C(q_j, p_j)$:[7]

$$\begin{aligned} & Min_{\tau,x_j} \quad p_j' \cdot x_j \\ & s.t. \quad Q\tau \geq q_j \\ & \quad x_j \leq X\tau \\ & \quad \tau \in \mathrm{R}^J_+ \\ & \quad \textstyle\sum_j \tau_j = 1 \end{aligned} \tag{2}$$

The overall cost efficiency measure is given by $p_j' \cdot x_j^* / p_j' \cdot x_j^0$, where x_j^* is the solution to the cost minimization problem in (2) and x_j^0 is the observed input vector for the jth firm.

4 Parametric efficiency results: sources and importance

Although many earlier studies of bank efficiency have sought to explain observed differences in total costs, greater accuracy should be obtained by separating total cost into operating and funding cost components as well as distinguishing between savings and commercial banks.[8]

4.1 Operating Cost Efficiency (DFA)

Operating cost (OC) includes the cost of labor, physical capital, and materials and is composed of three sets of influences:[9]

[3] Using U.S. banking data, DeYoung (1997) devised a test to determine how many years of separate cross-section regressions may be needed to have the random error likely average out close to zero and achieve a stable measure of efficiency. Six years was the result. We have 10 years of data and, instead of positing that measured efficiency should be stable, we interpret our results as an average indicator of efficiency over our period.

[4] The ratio $\bar{u}_{\min}/\bar{u}_i = \left(TC_{\min}/C(Q,P)_{\min}\right)/\left(TC_i/C(Q,P)_i\right)$ and when evaluated at the same output level and input prices, the predicted values of total cost $C(Q,P)_{min}$ and $C(Q,P)_i$ are equal as both are at the same point on the estimated total cost curve, leaving the ratio TC_{min}/TC_i . EFF can vary from zero (where bank i uses multiple times the resources of the most efficient bank) to one (where bank i is just as efficient as the most efficient bank).

[5] The level of inefficiency (INEFF) at the ith bank is INEFF = (1 - EFF)/EFF = (1/EFF) – 1.

[6] We thank an anonymous referee for the comments on this section.

[7] The nonparametric Malmquist index was not used here as it is not well-adapted to available banking data. The Malmquist index typically relates categories of balance sheet output (assets) to inputs (liabilities) plus labor and physical capital. This works well if outputs and inputs are actual quantities but in banking these are nominal or deflated values (using the same deflator for everything, not actual prices). Due to the balance sheet constraint, the sum of asset outputs always equals the sum of liability inputs so efficiency is equivalent to a simple ratio of labor and capital inputs to asset value. Other measurement problems also exist (c.f., Lozano-Vivas and Humphrey 2002).

[8] The data are already separated along these dimensions and aggregation to obtain total cost or reflect all banks unnecessarily restricts the separate costs or different banks to have the same (average) efficiency response. Spanish savings banks are similar to mutual organizations and are managed by depositors and provincial/local government entities while commercial banks are privately (stockholder) owned. This has led to differences in internal goals associated with service provision and contribution to local communities.

[9] Two intercepts are specified as the cross-section data used in each of the 10 separate regressions (one for each year) consists of two (pooled) semi-annual data periods. Specifying a single intercept would lower by three percentage points the efficiency value reported below for Technical Influences—the cost function. All the other operating cost efficiency values, however, are unchanged at the two digit level.

$$\ln OC = a_{01} + a_{02} + \text{ External } + \text{ Technical} + \text{ Internal} + \ln u + \ln v \quad (3)$$

Current external influences on a bank's operating costs may concern its asset size (Q_{TA}) to reflect differences in operating economies among banks,[10] the prevailing average wage in the region a bank is in (*WAGE*) which affects the average wage a bank pays for most of its workers, an index of property cost in the region (*IPP*) which will influence bank property costs, an indicator of regional business conditions reflected in the level of regional GDP (*GDPR*) which affects branch staffing requirements and ATM usage, a measure of asset market share (*MKSH*) to reflect the potential degree of local market power in the input market, and an indicator variable for the region in which the bank is located (*REGION*). Thus External influences on operating efficiency in (3) are:[11]

$$\begin{aligned}\text{External} = {} & e_1 \ln Q_{TA} + e_{11}0.5(\ln Q_{TA})^2 + e_2 \ln WAGE \\ & + e_3 \ln IPP + e_{22}0.5(\ln WAGE)^2 \\ & + e_{33}0.5(\ln IPP)^2 + e_{23}(\ln WAGE)(\ln IPP) \\ & + e_4 \ln GDPR + e_5 MKSH + e_6 REGION. \quad (3.1)\end{aligned}$$

The technical or cost function influences on operating cost follows a translog specification reflecting the transformation of banking inputs into outputs. Our two major banking outputs are loans (*LOAN*) and securities (*SEC*).[12] Input prices reflect each bank's actual average unit cost of labor (*PL*) while the ratio of depreciation to the value of physical capital approximates unit capital cost (*PK*). Materials prices are approximated by the opportunity cost of funds (*PM*—a market rate of interest).[13] This specification gives:

$$\begin{aligned}Technical = {} & a_1 \ln LOAN + a_2 \ln SEC + a_{11}0.5(\ln LOAN)^2 \\ & + a_{22}0.5(\ln SEC)^2 + a_{12}(\ln LOAN)(\ln SEC) \\ & + b_1 \ln PL + b_2 \ln PK + (1-b_1-b_2)\ln PM \\ & + b_{11}0.5(\ln PL)^2 + b_{22}0.5(\ln PK)^2 + ((b_{11}+b_{12}) \\ & + (b_{12}+b_{22}))0.5(\ln PM)^2 + b_{12}(\ln PL)(\ln PK) \\ & + (-b_{11}-b_{12})(\ln PL)(\ln PM) + (-b_{12}-b_{22}) \\ & (\ln PK)(\ln PM) + d_{11}(\ln LOAN)(\ln PL) \\ & + d_{12}(\ln LOAN)(\ln PK) + (-d_{11}-d_{12})(\ln LOAN) \\ & (\ln PM) + d_{21}(\ln SEC)(\ln PL) + d_{22}(\ln SEC) \\ & (\ln PK) + (-d_{21}-d_{22})(\ln SEC)(\ln PM). \quad (3.2)\end{aligned}$$

Internal influences on operating costs concern cost differences among banks that are mostly under the control of the banks themselves. This includes the number of ATMs and branch offices (*BR*) to provide for depositor convenience, and their mix (*ATM/BR*), which represents an important component of operating expenses. Also, a high ratio of loans to assets (*LOAN/TA*) should raise operating expenses as loans are more costly to produce than holding securities in a bank's portfolio. Finally, two Internal influences are approximate indicators of banking productivity such as a low ratio of labor per branch office (*L/BR*), saving labor costs but affecting service provision, and a high ratio of deposits per office (*DEP/BR*), reflecting greater output per capital input. The specification of Internal influences on operating costs in (3) is:

$$\begin{aligned}Internal = {} & i_1 \ln ATM + i_2 \ln BR + i_{11}0.5(\ln ATM)^2 \\ & + i_{22}0.5(\ln BR)^2 + i_{12}(\ln ATM)(\ln BR) \\ & + i_3 ATM/BR + i_4 LOAN/TA + i_5 L/BR \\ & + i_6 DEP/BR. \quad (3.3)\end{aligned}$$

The contribution of External, Technical, and Internal influences in (3) on overall efficiency, inefficiency, and the average absolute value of the residual as a percent of actual operating cost, are shown in Table 1.[14] Our preferred model combines all three influences on savings and commercial banks separately but they are also shown additively for illustration. Considering only Technical influences, the

[10] While past managerial decisions can affect bank size, in practice the vast majority of banks only grow slowly as their (externally influenced) market expands. The exception concerns those few banks that merge in a given year during which time inefficiency may improve (if costs are cut) or worsen (as back office integration problems arise). Consultant and other studies suggest that the net effect on the average bank merger on cost efficiency is close to zero (Rhoades 1993).

[11] Our model specifications do not always follow a standard second order Taylor series expansion but instead reflect judgments about whether a relationship is likely to be quadratic versus linear and whether or not interaction relationships are likely to be important.

[12] Unfortunately, publicly available data in Spain do not permit the specification of different types of loans.

[13] The market interest rate is a constant for all banks in each 6-month period. However, our 10 separate annual cross-section regressions are composed of two 6-month periods so this variable is not a constant in each regression run.

[14] Equations (3.1), (3.2), and (3.3) are alternative explanations of bank operating cost and, since the dependent variable (ln *OC* in Eq. (3)) is the same, these three equations are not a system of equations amenable to system estimation (so OLSQ is used). Cross-section estimation of the cost function (3.2) for each year for commercial (savings) banks yielded 20 (15) positive values for marginal costs for loan and security outputs. The 5 negative marginal costs were for securities, which is not surprising since the incremental cost of changing bank security holdings is close to zero in practice (all that is needed are a few traders, a small room, and phones).

Table 1 Bank operating cost efficiency—DFA, 1992–2001

Operating cost equation	EFF	INEFF	% Unexplained
Technical Influences	0.68	0.50	11.7%
External+Technical	0.72	0.41	8.6%
External+Technical+Internal	0.89	0.13	4.3%
Savings Banks			
External+Technical+Internal	0.94	0.07	1.9%
Commercial Banks			
External+Technical+Internal	0.96	0.04	1.5%

DFA efficiency results were not truncated in order to be comparable to the DEA method

usual case in the literature, efficiency for Spanish banks is low at EFF = 0.68, inefficiency is large at 0.50, and the average of unexplained operating cost is 11.7%. Putting External and Technical influences together raises efficiency to 0.72 and lowers inefficiency to 0.41. Combining all three influences together raises EFF to 0.89, reduces inefficiency to 0.13, and lowers average unexplained operating cost to only 4.3%.[15]

Still further increases in efficiency are obtained if the assumption of a common efficiency frontier is dropped and savings banks are separated from commercial banks. Here, efficiency is very high—between 0.94 and 0.96 and inefficiency is quite low at 0.04 to .07 while the average amount of unexplained operating cost is in both cases less than 2%.[16] Finally, although not shown here, signs on the estimated parameters show that operating cost falls and efficiency rises when banks deliver services using more ATMs versus branches (as expected) and also when each branch generates more deposits with less labor input. Selecting the ''right'' variables to explain differences in operating cost efficiency across banks apparently succeeds in reducing unexplained inefficiency to very low levels.[17]

4.2 Interest Cost Efficiency (DFA)

Looking at all sources of efficiency for savings and commercial banks separately, our preferred model, interest cost (IC) is specified as:[18]

$$\ln IC = a_0 + \text{External} + \text{Technical} + \text{Internal} + \ln u + \ln v. \quad (4)$$

Short-term External influences on a bank's interest or funding costs may concern its asset size (Q_{TA}) as larger banks often have a greater share of low cost deposits due to their large branch networks, the three-month market interest rate (*INTRATE*),[19] regional business conditions reflected in the level of regional GDP (*GDPR*) which can affect deposit availability and growth, asset market share (*MKSH*) to reflect the potential degree of market power in the deposit market, and an indicator variable for the region in which the bank is located (*REGION*). External influences on efficiency in (4) are thus specified as:

$$\begin{aligned} \text{External} = {} & e_1 \ln Q_{TA} + e_{11} 0.5 (\ln Q_{TA})^2 + e_2 \ln INTRATE \\ & + e_3 \ln GDPR + e_{33} 0.5 (\ln GDPR)^2 \\ & + e_{23} (\ln INTRATE)(\ln GDPR) + e_4 MKSH \\ & + e_5 REGION. \end{aligned} \quad (4.1)$$

The technical or cost function influences on interest cost follows the translog cost function specification above where the two major banking outputs are loans (*LOAN*) and

[15] The full DFA model contains 34–35 parameters. With semi-annual data on 46 (31) savings (commercial) banks, this gives 92 (62) observations for each year's separate regression and 58 (27) degrees of freedom. Similarly, there is no problem of an insufficient number of observations for our DEA model.

[16] Adding off-balance sheet (OBS) activities as a third banking output increased savings and commercial bank efficiencies by 1 percentage point (to 0.95 and 0.97, respectively) with a corresponding 1 percentage point reduction of inefficiency. As the effect was small, OBS activities were not added to the DEA model. Lagging by 6-months all but the ATM and BR variables in (3.3) reduced efficiency by 5 percentage points. However, decisions to alter loan and deposit rates do not generate a significant operating cost response since rate changes by one bank are typically matched by others and the small change in loan and deposit values is easily handled with the same number of branches and workers. Also, the time lag between decisions to add ATMs and branches and when they become operational is closer to 6 months for ATMs—the frequency of our data—and 9 months or more for branches (with little contemporaneous correlation).

[17] The DFA model estimates a separate regression for each year so each bank's residual for each year is determined by a different set of estimated parameters. Pooling the data and estimating a single set of parameters for all years with which to calculate each bank's yearly residual may or may not have much effect on the results. For savings banks in Table 1, EFF was 0.94 with yearly estimation and only falls to 0.92 using pooled estimation. For commercial banks, the reduction was greater (falling from 0.96 in Table 1 to 0.89 with pooled estimation).

[18] In contrast to the operating cost specification (3), two intercepts—one for each six-month period—is redundant here. The *INTRATE* variable in the External Influences Eq. (4.1) contains constant interest rates for each six-month period and reflects already our use of two periods for each annual cross-section regression. The exception is when Technical Influences (4.2) is separately estimated but here a two intercept specification yielded efficiency estimates that were the same at the three digit level.

[19] The level of market interest rates, through a yield curve, helps to determine bank funding costs. It is a constant for all banks for each 6-month period but varies within each annual cross-section regression (which covers two 6-month periods) and reflects an important influence on the level of bank deposit/loan rates from year-to-year.

securities (*SEC*) along with the actual average cost of funding (*PF*):

$$\begin{aligned}\text{Technical} &= a_1 \ln LOAN + a_2 \ln SEC + a_{11} 0.5(\ln LOAN)^2 \\ &\quad + a_{22} 0.5(\ln SEC)^2 + a_{12}(\ln LOAN)(\ln SEC) \\ &\quad + b_1 \ln PF + b_{11} 0.5(\ln PF)^2 + d_{11}(\ln LOAN) \\ &\quad (\ln PF) + d_{21}(\ln SEC)(\ln PF). \qquad (4.2)\end{aligned}$$

Lastly, we specify three measurable potential Internal influences on funding costs. A high ratio of ATMs to branch offices (*ATM/BR*) is believed to provide more convenience to depositors and, as a result, may permit a bank to pay a slightly lower deposit rate. In contrast, a high ratio of loans to assets (*LOAN/TA*) can bring in more revenue per deposited euro and so permit a bank to pay a higher deposit rate. Finally, a high ratio of deposits to assets (*DEP/TA*) can generate a lower average cost of funds for a bank as deposits are—depending on the interest rate cycle—often a lower cost source of funds than are other sources of borrowed money. As there seems to be no reason for a possible quadratic relationship here, the specification of Internal influences on interest costs is quite simple:

$$\text{Internal} = i_1 ATM/BR + i_2 LOAN/TA + i_3 DEP/TA. \qquad (4.3)$$

The contribution of potential External, Technical, and Internal influences in Eq. (4) on overall efficiency, inefficiency, and the average absolute value of the residual as a percent of actual interest cost, are all shown in Table 2. The usual cost function approach (Technical Influences in the table) yields an efficiency level of EFF = 0.91, inefficiency of 10%, so that only 2.2% of interest cost remains unexplained. This low level of inefficiency is due to the fact that the average price of funding (*PF*) ''times'' the value of assets needed to be funded (loans plus securities) explains almost all of the variation in interest cost across banking firms. As Spanish savings and commercial banks evidence a high level of interest cost efficiency with only a standard cost function specification, inefficiency measured by combining operating and interest expense into a single measure of total cost—the usual procedure in published studies—mostly reflects operating (not funding) inefficiencies.

Table 2 Bank interest cost efficiency—DFA, 1992–2001

Interest cost equation	EFF	INEFF	% Unexplained
Technical Influences	0.91	0.10	2.2%
External+Technical	0.922	0.085	1.93%
External+Technical+Internal	0.989	0.011	0.16%
Savings Banks			
External+Technical+Internal	0.999	0.001	0.04%
Commercial Banks			
External+Technical+Internal	0.993	0.007	0.17%

Combining External and Technical influences, measured efficiency is higher at 0.922 and the percent of unexplained interest cost across banks is quite small (1.93%). Due to collinearity between the variables in these two sets of influences on funding efficiency, the incremental improvement over considering just Technical influences alone—where EFF = 0.91—would be expected to be small. Finally, putting all three sets of influences together, efficiency rises to .989, inefficiency is only 0.011, and almost all the variation in interest cost among banks is explained (as the average percent that is unexplained falls to a minuscule 0.16%).

Use of a common cost frontier between savings and commercial banks is valid if there are few differences in efficiency between savings banks and commercial banks when estimated with a common versus separate cost frontiers. In our case there is almost no difference in interest cost efficiency at savings banks (at 0.999) compared to commercial banks (at 0.993) using separate frontiers compared to when a common frontier is applied (0.989, see Table 2).[20] For interest expenses, the funding price signals seen by managers are strong and clear enough to generate a very similar response, leading to little difference in average efficiency among banks. This is different from operating cost efficiency where there is more scope for management to hold alternative views and make different decisions that affect efficiency (e.g., altering the mix of ATMs to branch offices or hiring more or less labor per office to meet peak service demand, which affects operating costs, versus largely matching funding maturities with asset maturities and then minimizing interest expenses).

4.3 Parametric efficiency results

Our parametric results suggest the following conclusions. First, managerial ''control'' over cost efficiency, as evident from the role played by measurable Internal influences, is only really relevant for operating cost—not interest expense. Second, while the cost efficiency literature typically considers only Technical or cost function influences when determining banking efficiency, it is clear that augmenting this information with External influences—as pioneered by

[20] When a common frontier is specified, the parameters for savings and commercial banks are the same while when separate frontiers are used, these parameters can vary between these two sets of banks. If the efficiency values are quite similar, this means that allowing for different parameter values between savings and commercial banks (separate frontiers) has little or no economic significance (even if the parameter estimates may be significantly different in a statistical test).

Dietsch and Lozano-Vivas (2000) and Berger and Mester (1997)—yields a more accurate and higher level of measured efficiency.[21] Third, following earlier work by Frei et al. (2000) with micro measures of bank process productivity, much of the previously unexplained differences in bank cost efficiency can be accounted for using measures reflecting a bank's intensity of labor usage per branch office (the *L/BR* ratio), the ability to maximize deposits per branch office (a *DEP/BR* ratio), and decisions on how best to deliver services to depositors (the *ATM/BR* ratio).[22]

By achieving efficiency levels of over 0.99 for interest costs and from 0.94 to 0.96 for operational expenses, it is clear that banks do not actually misuse 20–25% of their resources. While a portion of these productivity differences among banks may be inadvertent and reflect a missed opportunity to reduce costs to some degree, it is also the case that many banks will purposely hire more workers per branch office and/or provide what seems to be ''too many'' ATMs and standard branch offices as part of their competitive strategy to be more accessible and provide more convenient services.

Finally, the remaining amount of unexplained efficiency differences is so low it is possible to argue that it could simply reflect the result of managerial decisions with a priori uncertain outcomes. It is possible to guess ''wrong'' about loan credit risks and so incur higher than expected loan monitoring and workout expenses. It is also possible to consistently lag other banks in adopting new technology or service offerings. Banks can also misjudge the likely growth of the local deposit market and provide too many branch offices or ATMs as well as place them in less productive locations compared to competitors. In this regard, once the major components of what was previously called inefficiency are identified, it would be interesting to determine the reasons for their variation. Are they the result of a conscious business strategy or an a priori management mistake? Answers here would help explain why cost efficiency differences among some banks seem to be persistent.

5 Nonparametric efficiency results: sources and importance

A more complete DEA model is also developed to identify the various sources of efficiency differences among banks. The formulation is the same as (2) with the addition of additional constraints $Z_\tau \geq Z_0$ that reflect similar external and internal influences on efficiency as were used above in the parametric operating and interest cost models:

$$
\begin{aligned}
& Min_{\tau,x_j} \quad p_j' \cdot x_j \\
& s.t. \quad Q\tau \geq q_j \\
& \quad x_j \leq X\tau \\
& \quad Z_\tau \geq Z_0 \\
& \quad \tau \in \mathrm{R}_+^J \\
& \quad \textstyle\sum_j \tau_j = 1
\end{aligned} \tag{5}
$$

The contribution of Technical, External and Internal influences on operating efficiency are presented in Table 3. With only technical influences, efficiency is already high at EFF = 0.95. Adding in External influences raises efficiency to 0.96 while including all three influences yields an efficiency value of 0.97. Eliminating the common efficiency frontier results in EFF = 0.98 for savings banks and EFF = 0.99 for commercial banks. In both cases, the full operating efficiency model yields residual inefficiency of 3% or less. Overall, the DEA approach to efficiency measurement gives results similar to those found using the parametric DFA approach. Importantly, when both approaches include similar External, Technical, and Internal influences on efficiency measurement, the level of unexplained or residual inefficiency is typically very low—much lower than values commonly reported in this literature.

The results for interest efficiency are shown in Table 4. Using only Technical influences, the level of interest efficiency is EFF = 0.87 which rises to 0.94 when External and Technical influences are combined. Adding Internal influences to this model gives EFF = 0.94. Finally, removing the assumption of a common frontier, interest efficiency at savings banks rises to 0.97 while that at commercial banks is 0.93. The small (3%) level of inefficiency for savings banks is similar to that from the full parametric model but inefficiency at commercial banks (8%) is larger.

6 Efficiency results and confidence intervals using the bootstrap technique

Confidence intervals for DFA and DEA approaches to efficiency measurement can be obtained using a bootstrap procedure involving multiple re-sampling.[23] For the DFA

[21] Other studies following this path are Lozano-Vivas et al. (2002), and Maudos et al. (2002).

[22] As one of the purposes of efficiency or frontier analysis was to make efficiency/productivity comparisons using a single overall measure rather than rely on a set of sometimes conflicting partial indicators, this result is largely expected, although its importance was unknown.

[23] Unfortunately, DEA estimates are serially correlated with unknown dependency among them. Consequently, we incorporate environmental variables on efficiency analysis using stage inference procedures that should solve this problem. In particular, we apply the Simar and Wilson (2007) algorithm. This algorithm is a coherent data generating process that allows environmental variables to influence efficiency. This model is estimated using a two-stage semiparametric bootstrap procedure.

Table 3 Bank operating cost efficiency—DEA, 1992–2001

Operating cost equation	EFF	INEFF
Technical Influences	0.95	0.05
External+Technical	0.96	0.04
External+Technical+Internal	0.97	0.03
Savings Banks		
External+Technical+Internal	0.98	0.02
Commercial Banks		
External+Technical+Internal	0.99	0.01

Annual DEA cross-section results were averaged to be comparable with DFA

Table 4 Bank interest cost efficiency—DEA, 1992–2001

Interest cost equation	EFF	INEFF
Technical Influences	0.87	0.15
External+Technical	0.94	0.06
External+Technical+Internal	0.94	0.06
Savings Banks		
External+Technical+Internal	0.97	0.03
Commercial Banks		
External+Technical+Internal	0.93	0.08

approach, this is much simpler than directly computing confidence intervals applying an asymptotic standard error formula using the estimated regression coefficients and their associated variance covariance matrix across an average of 10 separate yearly estimations. While the DEA approach assumes random error is zero, so our reported values here are presumed to be exact, it is still of interest to see the size of a confidence interval that would apply if the assumption of zero error was not met for this method. Finally, by comparing the bootstrapped confidence intervals for the DFA and DEA approaches, it is possible to determine the degree of overlap in efficiency results for these two methods.

Table 5 presents the bootstrapped mean values and 95% confidence intervals for the DFA and DEA efficiency approaches. The means and standard deviations were estimated from a distribution obtained from repeated sampling (with 10,000 replications) for bank-specific EFF results from both models. Three conclusions stand out. First, the mean efficiency values from the bootstrap procedure are all either identical to or within one percentage point of (after rounding) those reported in Tables 1–4 for the same set of EFF estimations. Second, the 95% confidence intervals about these mean values are tight and suggest that the mean values have low variance. Third, the confidence intervals are so tight that none of them overlap. This indicates that while some of the mean efficiency values are close together as point estimates, they also appear to be significantly different.

7 Differences in operating cost across banks (inefficiency) versus changes over time

The use of ATMs versus branch offices to deliver certain banking services (mainly cash acquisition, balance inquiry, and account transfers) as well as staffing decisions that affect the number of workers per branch office and the level of deposits raised per office[24] are seemingly important sources of previously unexplained cost efficiency differences. Other research has shown that, along with the substitution of low cost electronic payments (cards and electronic giro transactions) for paper-based instruments (checks and paper-based giro transfers), changes in service delivery methods in Spain and across 12 European countries have contributed to a large reduction in bank operating cost during the 1990s.

The observed ratio of operating cost to total assets—an indicator of unit operating expense across banks—fell by 35% over 1992–1999 in Spain (Carbó-Valverde, et al. 2006) and by 24% for 12 European countries over 1987–1999 (Humphrey, et al. 2006). The reduction in the average operating cost to asset ratio for savings and commercial banks in Spain is seen in the density functions of this ratio in Figs. 1 and 2 over 1992–2001. The mean ratio for savings banks in 1992 fell by 31% relative to 2001. For commercial banks, the drop was 37% but the dispersion is fairly constant over this 10-year period. An approximate ''inefficiency'' dispersion measure is computed by averaging the deviation of each bank's operating cost to asset ratio from the lowest observed ratio separately for 1992 and 2001. The reduction in cost dispersion for savings banks over 1992–2001 was 6.3% of the mean operating cost to asset ratio in 1992 with a 1.1% rise for commercial banks. Thus the average savings bank experienced 4.9 times the reduction in unit operating cost over 1992–2001 than was obtained from the reduction in cost dispersion over the same period (from 0.31/0.063). For commercial banks, all of the reduction in unit operating cost over time came from time-series changes (since dispersion relative to the bank with the lowest ratio rose rather than fell).

These comparisons indicate that, for social and regulatory policy purposes, determining the source of banking cost changes over time is more informative than doing the

[24] This latter influence is more likely a result of previously locating branches in areas where incomes are relatively high than due to independent internal efforts by management at existing offices to raise deposits.

Table 5 Bootstrap results and confidence intervals, 1992–2001: DFA and DEA total, interest, and operating cost efficiency

	Mean		Confidence Intervals	
	DFA	DEA	DFA	DEA
Operating Cost				
Technical Influences	0.679	0.954	[0.645, 0.691]	[0.939, 0.958]
External+Technical+Internal	0.887	0.968	[0.873, 0.896]	[0.961, 0.987]
Savings Banks				
External+Technical+Internal	0.940	0.976	[0.929, 0.947]	[0.972, 0.995]
Commercial Banks				
External+Technical+Internal	0.959	0.989	[0.948, 0.967]	[0.981, 0.996]
Interest Cost				
Technical Influences	0.911	0.875	[0.908, 0.916]	[0.861, 0.887]
External+Technical+Internal	0.989	0.941	[0.988, 0.989]	[0.932, 0.954]
Savings Banks				
External+Technical+Internal	0.999	0.973	[0.998, 0.999]	[0.962, 0.979]
Commercial Banks				
External+Technical+Internal	0.993	0.932	[0.992, 0.993]	[0.926, 0.945]

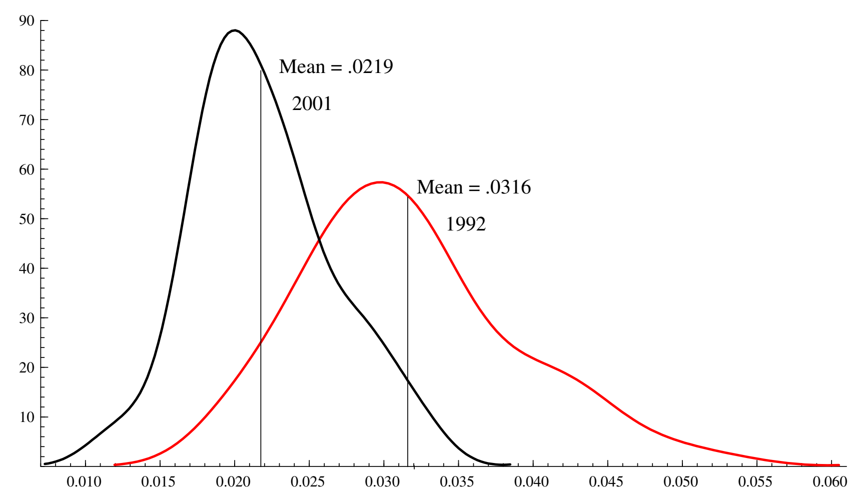

Fig. 1 Density: Savings bank operating cost/total asset ratio (1992 comparted to 2001)

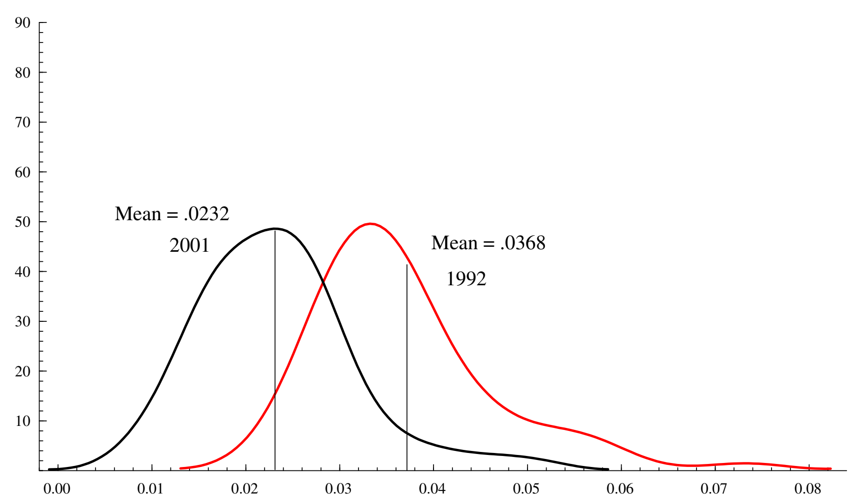

Fig. 2 Density: Commercial bank operating cost/total asset ratio (1992 comparted to 2001)

same among banks at a point in time (i.e., monitoring changes in efficiency). The fact that the same variables are the apparent source of unexplained efficiency as well as an important source of the dramatic reduction in bank operating costs in Spain and Europe over time, raises an interesting question: if banks can markedly reduce their level of unit operating costs over time, why are they seemingly unable to do much the same compared to other banks? Are the cost changes over time largely due to obvious trends or opportunities (e.g., using lower cost ATMs to minimize the need to expand branch offices, encouraging the use of cheaper electronic payments, adopting more automated credit risk procedures, etc.) while active implementation of efficiency benchmarking among banks represents less obvious fine tuning? The fact that the reduction in the banking industry's average operating cost over time was 5 times the apparent reduction from improved efficiency would be consistent with this possibility. More likely, however, differences in cost efficiency among banks are relatively stable over time because the dispersion and adoption of innovations is imperfect so that there will always be leaders and laggards even as the cost frontier shifts down over time (generally lowering all banks' costs but not having much effect on dispersion).[25]

8 Summary and conclusions

A recent survey of efficiency results from 130 studies covering 21 countries' banking sectors suggested that the average bank appears to experience total operating plus interest costs that are from 20% to 25% higher than the most cost-efficient institution (Berger and Humphrey 1997). At usual ratios of bank net income to total costs, such levels of inefficiency suggest that the average bank—not just the most inefficient among them—could more than double their profits and return on assets by restructuring their operations to resemble banks that appear to be most efficient. With such a strong incentive to change behavior, it is surprising that these levels of measured inefficiency do not seem to be falling over time.

We specified a fuller set of influences that could explain differences in cost efficiency among banks and, to obtain greater accuracy, total costs were separated into their interest and operating cost components. With a parametric model (Distribution Free Approach), unexplained inefficiency levels for Spanish savings and commercial banks averaged only 1–4% compared to the 20–25% levels commonly reported in the literature. Use of a non-parametric model (Data Envelopment Analysis) gave only 2–6% average inefficiency.

Our broader set of efficiency influences concerned external influences outside of the control of management, technical influences associated with transforming banking inputs into outputs within a cost function, and influences partly under managerial control and thus internal to the firm. While the cost efficiency literature typically considers only technical or cost function influences when determining banking efficiency, it is clear that augmenting this information with external influences (c.f., Dietsch and Lozano-Vivas 2000; Berger and Mester 1997) yields a more accurate and higher level of measured efficiency. We found that most of the previously unexplained differences in banking cost efficiency evidenced in other studies is actually associated with partial indicators of banking productivity such as a bank's intensity of labor usage per branch office (the *L/BR* ratio) and the ability to maximize deposits per branch office (a *DEP/BR* ratio), as well as decisions on how best to deliver services to depositors (the *ATM/BR* ratio). By achieving efficiency levels of from 0.94 to 0.96 for operational expenses and over 0.99 for interest costs with a parametric Distribution Free Approach model, it is clear that banks do not misuse 20–25% of their resources.[26] Examination of confidence intervals suggest that the DFA and DEA efficiency values are significantly different although both methods yield similar very high efficiency values when the full set of influences (external, technical, and internal) are incorporated in the analysis.

The same changes in service delivery methods that seem to explain previously unexplained efficiency differences among banks in Spain are also those that, along with the switch to lower cost electronic payments, have contributed to a dramatic reduction in bank operating cost in Spain and across 12 European countries during the 1990s. The observed ratio of operating cost to total assets—an indicator of unit operating expense across banks—fell by 35% over 1992–1999 in Spain. In contrast, the dispersion of bank unit costs was more stable. Indeed, the average savings bank experienced 4.9 times the reduction in unit operating cost over 1992–2001 than was obtained from the reduction in cost dispersion over the same period. For commercial banks, all of the reduction in unit operating cost over time came from time-series changes. Consequently, determining the source of banking cost changes over time is more informative than looking at efficiency changes among banks at a point in time even though the same set of cost influences appear to be at work in both cases. We close with a question: if banks can markedly reduce their level of unit

[25] This reference to the diffusion theory of innovation was suggested by a referee.

[26] With the non-parametric Data Envelopment Analysis model, operating cost efficiency was 0.98–0.99 while for interest cost it was 0.92–0.97.

operating costs over time, why are they seemingly unable to also markedly reduce the cost dispersion among banks? It may be that the fine tuning required to markedly reduce cost dispersion, which during 1992–2001 was at most less than 20% of the cost reduction actually achieved for the industry over time, gets lost in the process of achieving the five-fold industry-wide cost changes occurring concurrently. More likely, the diffusion of innovations is imperfect and some banks will always lag others and manifest itself as a persistent difference in cost efficiency.

Data Appendix

Data are observed semi-annually over 1992–2001, giving 1,540 panel observations. The data set includes all savings banks, all but the very smallest commercial banks (which were excluded due to missing data), and no cooperative banks (who also had missing data). The sample covers 90% of total banking assets in Spain in 2001. Starting at the end of 2001, all data were backward aggregated to obtain the same number of banks with the same bank code in each year. If two banks had existed but merged before the end of the sample period, they are aggregated over the period they existed separately and so enter the data set as a single composite bank for the entire period. This permits the use of a balanced panel data set from which to compute the DFA average residual for each bank separately. Descriptive statistics are shown below.

Variable	Mean	Standard deviation
TC	524,826	1,306,915
LOAN	5,650,417	13,640,000
SEC	2,241,031	6,798,484
PF (interest rate)	0.048	0.028
PL (annual price)	44.725	44.766
PK	0.135	0.0162
IC	344,020	939,119
Q_{TA}*(ln of value)*	14.82	1.35
INTRATE=PM (percentage)	7.107	3.385
GDPR	46,980,000	29,130,000
MKSH (percentage)	0.012	0.03
ATM/BR	0.865	0.946
LOAN/TA	0.737	0.097
DEP/TA	0.884	0.075
OC	180,806	379,849
WAGE	1,217	181
IPP (index number)	120	36
ATM (number)	396	744
BR (number)	391	690
L/BR (number per branch)	8.14	13.3
DEP/BR (value per branch)	33,583	162,821

Note: Values shown are in 1,000 of euros, or ratios of these values, unless otherwise noted

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